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Solution by S. A. COREY, Hiteman, Iowa.

$$\int_0^{\frac{1}{2}\pi} d\theta \sqrt{[1 + \sin^2 \theta (1 - 4 \cos \theta)]} \equiv \int_0^{\frac{1}{2}\pi} d\theta \sqrt{(1\frac{1}{2} - \cos \theta - \frac{1}{2} \cos 2\theta + \cos 3\theta)} \dots\dots\dots (1).$$

The value of the second member of (1) is easily computed by aid of the formula

$$\begin{aligned} f(x) - f(0) = & \frac{x}{m \cdot 2} \left\{ [f'(x) + f'(0)] + 2 \left[f\left(\frac{x}{m}\right) + f'\left(\frac{2x}{m}\right) + f'\left(\frac{3x}{m}\right) + \dots\dots\dots \right. \right. \\ & \left. \left. + f'\left(\frac{m-1}{m}x\right) \right] \right\} - \frac{B_2 x^2}{m^2 \cdot 2!} [f''(x) - f''(0)] + \frac{B_2 x^4}{m^4 \cdot 4!} [f^{iv}(x) - f^{iv}(0)] + \dots\dots\dots \\ & \dots\dots\dots + (-1)^n \frac{B_n x^{2n}}{m^{2n} \cdot (2n)!} [f^{2n}(x) - f^{2n}(0)] + \dots\dots\dots \end{aligned}$$

$B_1, B_2, \dots\dots\dots$, being Bernoulli's numbers.

Taking $m=5$, and $n=2$, the value of the definite integral is found to be 1.17066. For greater accuracy, larger values of m and n may be taken.

II. Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

Let $\sin^2 \theta (1 - 4 \cos \theta) = x$. Then by expanding the expression we get

$$(1+x)^{\frac{1}{2}} = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 - \frac{5}{128}x^4 + \frac{7}{256}x^5 - \dots\dots\dots$$

$$\int_0^{\frac{1}{2}\pi} d\theta = \frac{1}{2}\pi = 1.5708; \quad \frac{1}{2} \int_0^{\frac{1}{2}\pi} x d\theta = \frac{1}{8}\pi - \frac{2}{3} = -.2740;$$

$$- \frac{1}{8} \int_0^{\frac{1}{2}\pi} x^2 d\theta = -\frac{11\pi}{128} = -.0699; \quad \frac{1}{16} \int_0^{\frac{1}{2}\pi} x^3 d\theta = \frac{35\pi}{512} - \frac{5}{256} = -.0194;$$

$$- \frac{5}{128} \int_0^{\frac{1}{2}\pi} x^4 d\theta = \frac{2}{75} - \frac{2975\pi}{32768} = -.0138;$$

$$\frac{7}{256} \int_0^{\frac{1}{2}\pi} x^5 d\theta = \frac{16401\pi}{131072} - \frac{54761}{137280} = -.0059;$$

$$\therefore \int_0^{\frac{1}{2}\pi} \sqrt{[1 + \sin^2 \theta (1 - 4 \cos \theta)]} d\theta = 1.1858 -.$$

Also solved by the Proposer.

MECHANICS.

170. Proposed by ELISHA S. LOOMIS, Ph. D., Berea, Ohio.

Two angles of iron, $A_1 CD$ and $A_1 CA_3$, move freely on a pivot at C . Rods $B_1 A_1$ and $B_1 A_3$ are attached respectively at A_1 , and at some point A_3 so that

when B_1 moves along the rod CR , which is perpendicular to A_1A_4 , CD and CA_3 shall coincide in position with CE which is perpendicular to rod KR . When angle A_1CD is 135° find CA_3 in terms of CA_1 .

Also find the following:

a). That value of CB_1 which will require least effort exerted at B_1 to cause CA_3 to take the position CA_4 .

b). That value of CB_1 which will cause B_2A_2 , if produced, to pass through the point A_1 .

c). As CB_1 varies in value, what is the locus of the intersection of A_1B_1 and A_2B_2 ? Of B_1A_3 and B_2A_4 ?

d). Suppose angle A_1CD to be any other angle than 135° , then find CA_3 in terms of CA_1 .

Solution by the PROPOSER.*

Determination of the value of CA_3 in terms of CA_1 when angle $A_1CD = 135^\circ$.

a). In triangle CA_1B_1 , let $CA_1 = r$ and $CB_1 = vr$. Then $A_1B_1 = r\sqrt{1+v^2}$.

b). In triangle CA_2B_2 , $CA_2 = r$, $A_2B_2 = r\sqrt{1+v^2}$, and angle $A_2CB_2 = 45^\circ$.

Then, by trigonometry, $\sin B_2 = \frac{r \sin C}{r\sqrt{1+v^2}} = \frac{1}{\sqrt{2}\sqrt{1+v^2}}$.

$$\therefore CB_2A_2 = \sin^{-1} \frac{1}{\sqrt{2}\sqrt{1+v^2}}.$$

$$\therefore CA_2B_2 = 135^\circ - \sin^{-1} \frac{1}{\sqrt{2}\sqrt{1+v^2}}.$$

$$\therefore CB_2 = \frac{r\sqrt{1+v^2} \sin \left(137^\circ - \sin^{-1} \frac{1}{\sqrt{2}\sqrt{1+v^2}} \right)}{\sin 45^\circ}$$

$$= r\sqrt{2}\sqrt{1+v^2} \sin \left(135^\circ - \sin^{-1} \frac{1}{\sqrt{2}\sqrt{1+v^2}} \right)$$

$$= kr, \text{ in which } k = \sqrt{2}\sqrt{1+v^2} \sin \left(135^\circ - \sin^{-1} \frac{1}{\sqrt{2}\sqrt{1+v^2}} \right).$$

c). In triangle CB_1A_3 , $CB_1 = vr$ and angle $B_1CA_3 = 135^\circ$. Draw A_3H perpendicular to CK . Then $CA = A_3H$. Let $CA_3 = xr$, and $CH = p = xr/\sqrt{2}$.

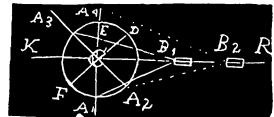
$$\begin{aligned} \therefore (B_1A_3)^2 &= (CB_1)^2 + (CA_3)^2 + 2CB_1 \times CH = (vr)^2 + (xr)^2 + \sqrt{2}xvr^2 \\ &= r^2(v^2 + x^2 + \sqrt{2}vx). \end{aligned}$$

d). In triangle B_2CA_4 , $CA_4 = CA_3 = xr$, $B_1A_3 = B_2A_4$, $CB_2 = kr$, and angle $B_2CA_4 = 90^\circ$. $(B_2A_4)^2 = (B_2C)^2 + (CA_4)^2 = r^2(k^2 + x^2)$.

$$\therefore r^2(v^2 + x^2 + \sqrt{2}vx) = r^2(k^2 + x^2). \quad \therefore v^2 + \sqrt{2}vx = k^2. \quad \therefore \sqrt{2}vx = k^2 - v^2.$$

$$\therefore x = \frac{k^2 - v^2}{\sqrt{2}v}. \quad \therefore xr = \frac{r \left[\left(\sqrt{2}\sqrt{1+v^2} \sin \left(135^\circ - \sin^{-1} \frac{1}{\sqrt{2}\sqrt{1+v^2}} \right) \right)^2 - v^2 \right]}{\sqrt{2}v}.$$

Also solved by G. B. M. Zerr.



*Dr. Loomis writes us that the determination of CA_3 in terms of CA_1 is needed in a certain mechanism for producing dissolving views in a magic lantern. Ed. E.